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Sound Science White Paper Series #01

Contingency Tables in Land Stewardship: Evaluating Differences among Categorical Variables

Land stewards often collect frequency data, counting observations and categorizing them according to specific attributes of interest. Count or frequency data for categorical variables are usually analyzed in contingency tables using Chi^2 tests.

Contingency table analyses are significantly underutilized in conservation and stewardship work. This is unfortunate because the approach can be used to analyze a wide variety of data: abundance of a species or event using nested frequency plots, vegetation cover using line-point intercept, or change in condition assessments among many observation points.

The principle benefits of contingency table analyses are:

- They are very flexible in how the data can be conceived and arranged.
- The analyses are relatively free of constraining statistical assumptions.
- The presentation is intuitive and easy to communicate.
- The calculations are straight forward and easy to learn.

This White Paper is a primer on displaying categorical monitoring data in contingency tables and calculating Chi^2 tests to evaluate differences among categorical variables.

Categorical and other types of data

Categorical variables are those in which observations are assigned to a category or group (**Figure 1)**. Such variables are also called “nominal” or “discrete”. Ordinal variables are similar in that they are based on categories, but they have a logical order (i.e., the groups themselves can be ordered). Both types of data are made up of the counts, or frequencies, in each category and both can be analyzed using contingency tables. A common categorical variable is simple presence/absence, but other categorical stewardship data include assigning observations to condition categories, or identifying trail segments as smooth/rough/hazardous/impassible.

Type of variable	Description	Examples
Categorical (also called Nominal or Discrete)	Discrete categories that have no inherent order	1. Color 2. Park unit 3. County
Ordinal	Discrete categories that can be ordered, or have a logical sequence in which certain classes are “greater or more than” others	1. Green/Amber/Red condition classes 2. Vegetative cover measured in 4 classes (<10%, 10-50%, 50-90%, >90%) 3. Daubenmeyer cover classes
Continuous	Variables without categories, in which any value is theoretically possible	1. % cover of vegetation 2. Density of woody seedlings 3. Number of visitors per natural area

Figure 1. Definitions for the three major types of measurement variables.

Overview of contingency table analyses

Contingency tables display the joint frequencies of two or more categorical variables in rows and columns: each frequency or count falls into two or more categories. Then Chi^2 analyses test for correlations among the variables. When one of the variables is time (e.g., year of sampling) this approach tests for correlations between time and other variables. In other words, this tests for change over time, or trend. Examples of contingency table data and management questions that could be addressed include:

- Counts of three species on each of five natural areas to test for correlations between site and abundance of particular species.
- Counts of erosion events on road segments every two years to test for changes in road segment condition over time.
- Counts of Visitor Areas in Green/Amber/Red condition categories in each of five years to test for changes in condition over time.
- Counts of camp sites in one of five different condition categories near the preserve entrance vs. in remote sections to test for differences in condition of camp sites in different locations.
- The abundance, based on frequency plots, of a noxious weed in areas with different land use to test for correlations between land use and abundance of noxious weeds.
- The number of locations along line point-intercept transects that are bare ground over three time periods to test for changes in bare ground over time.

- The frequency of three condition classes (fully functional, partially functional, dysfunctional) for each of five different types of land restoration BMPs in order to compare effectiveness of different BMPs.

Figure 2 is an example of a contingency table containing the frequency of gullies in four different zones over three monitoring periods. **Figure 3** is an example of a contingency table used to assess changes in condition in a single camping area over three years.

	Zone A	Zone B	Zone C	Zone D
2004	4	6	1	0
2006	12	11	1	1
2008	26	9	4	2

Figure 2. The frequency of significant erosion gullies (> 1m deep and 1m wide and 10m long) in five Zones over three monitoring periods.

	Green	Amber	Red
2004	20	60	20
2005	25	55	20
2006	45	40	15

Figure 3. The frequency of observation points classified as green, amber, or red (GAR) in Camping Area Johnson. GAR is a composite score of several other observations, including erosion events and vegetative cover.

In general, contingency table analyses test whether data in the rows and columns are independent. In **Figure 2** the analysis asks whether the differences in the frequencies of gullies among years depends on the Zone; in other words, does the pattern of change among years vary among Zones? In **Figure 3** the analysis asks whether the relative frequencies among GAR categories depends on year (that is, are there more Green areas in later years?).

See the section **Web Resources** at the end of this paper for a list of Internet resources that discuss contingency table analysis and perform contingency table calculations.

Analysis of contingency table data

Contingency table data are based on categorical or ordinal variables (i.e., observations are assigned to categories) and usually do not conform to the assumptions of normal parametric statistics. For example, the underlying observations that make up frequency data do not typically exhibit a "normal" or bell shaped distribution. Consequently t-tests, ANOVA, regression and other classic analyses based on these assumptions generally are not appropriate for these data. Contingency table analyses are classified as "non-parametric" analyses, as they are able to analyze data that do not meet the assumptions of parametric analyses.

> [Note that although frequency data are typically not normally distributed, in some cases these data can be transformed and analyzed with parametric statistics. For example, line-point intercept data are often summarized (i.e., "rolled up") by transect to create continuous variables (variables that can take on any value within a specified range, for example between 0 and 100%). In such cases transects become units of observation, which significantly reduces the sample size. This method is both useful and problematic, but is beyond the scope of this paper.]

Contingency table analyses test the statistical significance of associations among categorical variables by examining the difference between the observed frequencies and the frequencies one would anticipate if there were *no relationship* among the variables, which are called the expected frequencies. The basic hypothesis posed in contingency tables is: Are the frequencies in the rows independent of the frequencies in the columns? There are extensive discussions of the theory behind contingency analyses in Zar (1996), Sokal and Rohlf (1986) or virtually any other textbook of basic statistics.

There are many of tests of association among frequency variables. Common ones are:

- Chi^2 (for independent samples)
- G test
- Goodness-of-fit tests for N x 1 tables
- Binomial test
- Fisher's Exact Test (a Chi^2 test for tables that have low cell frequencies)
- McNemar Chi^2 (a Chi^2 for repeated observations in permanent plots)

Chi^2 is the typical statistic for such analyses although Chi^2 and the G test are largely interchangeable. The G statistic (sometimes called the Log-ratio test) is also commonly used and is sometimes preferable in tables that are larger than two dimensions (i.e., three or more categorical variables, for example Condition by Area by Time). Chi^2 is preferred when total frequency is less than 100. All of the tests are similar in that they compare expected to observed frequencies.

In this paper the Chi^2, G test for goodness-of-fit, Fisher's Exact test and McNemar are examined.

Displaying Results

In addition to tables of frequencies, bar charts and pie charts are effective ways to display frequency data. For example, **Figure 4** displays the data in **Figure 3** as a cumulative bar chart.

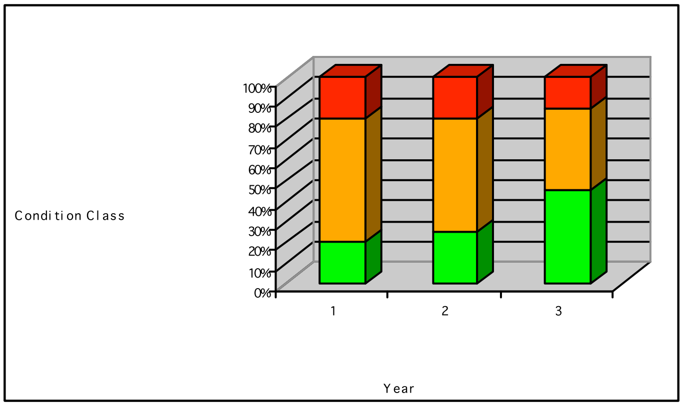

Figure 4. The data from Figure 3 displayed as a bar chart. This chart was created in MS Excel.

Confidence Intervals

As with means and other averages, confidence intervals can help describe the level of uncertainty in data. One can calculate CI's for cell frequencies: a 95% CI around a frequency represents the upper and lower limits within which one is 95% certain the <u>true</u> population frequency falls. Confidence intervals for frequency data are based on a binomial, and not normal, distribution. Thus, the formulas for calculating CI's for continuous data (data that can take on any value within a specified range) are not appropriate for frequency data. See the **Web Resources** section at the end of this paper for web sites that discuss and calculate confidence intervals for frequencies and proportions.

Independence of Samples

Although contingency table analyses are nonparametric analyses, they are not entirely without assumptions. For example, a central assumption of Chi^2 analyses is the <u>independence of observations</u>. The most important element of this is that each observation point can be recorded only once. Permanent plots, where data are collected repeatedly over time, violate this assumption because subjects (i.e., plots) contribute errors more than once to the calculation.

Therefore,

- For simple tests of association (frequency data within years): Chi^2
- For tests of association among years, temporary plots: Chi^2
- For tests of association among years, permanent plots: McNemar test

An Example of the Arithmetic of Chi^2

Figure 5 displays data from 352 temporary plots over two years (n=176 in each year). The plots were examined for the presence or absence of Species X.

	Year 1	Year 2	Total
Plots containing Sp. X	130	132	262
Plots without Sp. X	46	44	90
Total	176	176	352

Figure 5. An example of a 2x2 contingency table for calculation of a Chi^2 test. The bottom row and the right column are called the "marginal" frequencies. The bottom right cell is the Total Frequency.

Remember that contingency table analyses test whether data in rows and columns are independent by examining the difference between observed and expected frequencies. If Year (columns) has no effect on presence/absence of Species X (rows) (i.e., Year and Species X occurrence is independent), then one would expect the proportion of plots with and without X to be the same. The best guess of the proportion of plots containing Species X is based on the marginal frequency (located in the 'Total' column located on the edge, or margin, of the table) of Species X and the total number of plots: 262/352 = 74.4%. That is, of all plots (n=352) 74.4% contain Species X. So, the expected number of plots with Species X in each of the two years is 176 (the number of plots each year) x 0.744 = 131. The expected number of plots each year without Species X is calculated in a similar way. The expected values for all cells in this example are shown in brackets in **Figure 6.** Now one can compare these expected frequencies (based on an assumption of no relationship between Years and Presence/Absence with the frequencies that were actually observed.

	Year 1	Year 2	
Plots containing Sp. X	130 *[131]*	132 *[131]*	262
Remaining Plots	46 *[45]*	44 *[45]*	90
	176	176	352

Figure 6. The same 2x2 contingency table as in Figure 4, but including the expected frequencies (in brackets).

Calculating the Chi^2 Statistic

Example 1

The Chi^2 statistic tests the statistical significance of the difference between the observed and expected frequencies.

The calculation of Chi^2 for the **Figure 6** data is:

$$\frac{(130-131)^2}{131}+\frac{(132-131)^2}{131}+\frac{(46-45)^2}{45}+\frac{(44-45)^2}{45}=0.06$$

Degrees of Freedom (df) are determined by the number of row cells (minus one) times the number off columns (minus one), not the total frequency. In this case df = (2-1) x (2-1) = 1.

The level of significance of the difference between observed and expected frequencies is determined by comparing the calculated value of Chi^2 to critical levels of significance in a Chi^2 table, based on the degrees of freedom (see **Appendix 1**).

The critical value of Chi^2 for p=0.05 and with one degree of freedom is 3.841. The calculated Chi^2 = 0.06, is smaller than the critical value. Thus, the conclusion from Figure 6 is that the observed frequencies do not deviate from expected, and that the presence/absence of Species X is independent of Year. In other words, there has been no change in the abundance of Species X from Year 1 to Year 2.

In addition to **Appendix 1**, an interactive lookup table for significance of Chi^2 values can be found at: http://www.fon.hum.uva.nl/Service/Statistics.html .

Also see the section **Web Resources** at the end of this paper.

Tip: At the following Internet site one can download an Excel-based calculator for Chi^2: http://udel.edu/~mcdonald/statchiind.html

Example 2

Suppose one is interested in estimating the frequency of bare ground using line point intercept data. The monitoring question is: Does the risk of erosion differ over two years? In this example the risk of erosion is approximated by the frequency of vegetated area and of bare ground (i.e., exposed soil). The data and analysis is shown in **Figure 7**.

The calculated Chi^2 = 20.8, which is larger than 3.841, the critical value of Chi^2 for p=0.001 and with 1 degree of freedom (**Appendix 1**). The actual probability** of getting such a Chi^2 if the true difference between years were zero is very small

(p =0.00005). Stated another way: the *expected* number of bare points in each year is 60. The chance of getting the observed frequencies (46 and 76) due to 'chance' (random sampling error) is small, and hence one concludes that the amount of bare ground has increased.

	Line Point Intercept			
Total Number of Points	176			
All Points Combined	Year 1	Year 2		
Points Hitting Veg	130	100	Chi Square	20.84211
Points hitting bare grd	46	76	*p* =	0.0000050

Figure 7. Example of a Chi^2 analysis of line-point intercept data. Has change occurred in the vegetated area between Year 1 and Year 2? The probability of obtaining these frequencies when the <u>true</u> frequencies are the expected frequencies is very small (p = 0.000005). Therefore, the conclusion is that yes, change has occurred: in this case there is significantly more bare ground in Year 2.

Note that statistical packages, such as the one that calculated the probability in **Figure 7, calculate the <u>exact</u> probability. The numbers in **Appendix 1** are "critical" values of Chi^2 <u>beyond which</u> significance is indicated.

Collapsing Categories to Improve Interpretation

A significant Chi^2 for more than two categories suggests that, among all categories, there are deviations from expected frequencies. It does not indicate <u>which</u> categories are different.

There are two solutions.

(1) One may perform analysis on subsets of categories to detect particular patterns. For example, examine the data from **Figure 3**. Imagine that a Chi^2 detected significant non-random association within this 3x2 table. The analysis does not indicate which condition classes are different (although it may be obvious). Collapse two categories into one (say, sum the frequencies of Amber and Green into one) and repeat the analysis on a 2 years x 2 condition (Red vs. not Red) table. This type of exploration may reveal the patterns in the data.

For example, Chi^2 results for the data in **Figure 3** are:

$Chi^2 = 16.44$
df = 4
p = 0.0025

This result suggests that Condition and Years are not independent (p=0.0025) and that the frequencies of the three condition categories depend on year. In other words, condition has changed over time. However, this result is silent on which years of Conditions differ. Visual inspection of the data appear to suggest that most of the change is due to conversions of Amber sites to Green.

Figure 8 collapses Amber and Red in a single category. Chi^2 results for the data in **Figure 8** are below, and suggest that the increased frequency of Green sites is statistically different from the pattern in non-Green sites:

$Chi^2 = 16.34$
df = 2
p = 0.00028

	Green	Amber+Red
2004	20	80
2005	25	75
2006	45	55

Figure 8. The data from Figure 3 but reclassified as green and amber or red.

Figure 9 is the data from **Figure 3** without Green. Remember that this analysis is not whether there are more Amber sites than Red; only whether the frequencies of the columns depend on the rows. A Chi^2 comparison of the frequencies in this table suggests that Condition (Amber and Red) is independent of Year.

$Chi^2 = 0.101$
df = 2
p = 0.951

	Amber	Red
2004	60	20
2005	55	20
2006	40	15

Figure 9. The data from Figure 3 without the Green points. This is done in order to compare the frequencies of Amber and Red.

(2) There are more sophisticated analyses available for categorical data called Log-linear models. These models can partition out sources of variance among various levels of different categories. These techniques are beyond the scope of this paper. For more information see (http://faculty.vassar.edu/lowry/abc.html).

Contingency Tables with Only One Row

Imagine that the data from **Figure 3** was from a single year and the question is whether the frequencies in different condition categories are equal. In this case the analysis is of a 3x1 table (**Figure 10**).

The appropriate analysis is a Chi^2 test for goodness of fit or G test for goodness of fit. An Excel based calculator can be found at http://udel.edu/~mcdonald/statgtestind.html

	Green	Amber	Red
2006	45 [33.3]	40 [33.3]	15 [33.3]

Figure 10. The data from **Figure 3**, however including only 2006. Expected frequencies under a hypothesis of random assortment are shown in brackets.

The expected frequencies under a hypothesis of random assortment are each 33.3. The results confirm that the observed frequencies differ from the expected:

G = 17.64
p = 0.00015

This analysis is not restricted to even probabilities among years. Specific hypotheses about relative frequencies can be tested.

The Limitations of Chi^2

- Chi^2 technically assumes no ordering of observations (i.e., ordinal data). However, Chi^2 can be applied to ordered classes.

- When the expected frequencies in some cells are small the test is inaccurate and erratic. One should use the Fisher Exact Test in these cases. As a rule of thumb, if more than 20% of cells have expected frequencies less than 5, use the Fisher Exact Test (below).

- Chi^2 assumes independence of data. When using permanent plots, use the McNemar's test (below).

Fisher's Exact Test – When Cell Frequencies are Very Low

Fisher's Exact Test is a useful replacement for Chi^2 when expected cell frequencies are very small. This is a common situation when the events being described are rare. For example, if one is trying to associate the frequency of severe injuries to soldiers in military training lands – a rare occurrence – in different maneuver areas a Fisher Exact Test will typically produce more reliable analysis than a Chi^2.

Tip: When more than 20% of cells have frequencies of five or fewer (and therefore a Fisher's Exact Test is needed), consider changing the data collection method or rescaling the data (i.e., lumping categories) so that the frequencies are not so low.

Hand calculation of Fisher Exact Test is tedious and beyond the scope of this paper, but the test is useful in RTLA work. See the section **Web Resources** at the end of this paper for a list of Internet sites that calculate Fisher Exact Test.

Figure 11 shows hypothetical data for serious injuries over two time periods. The low frequency of injuries makes standard Chi^2 problematic. Fisher's Exact Test suggests that the number of injuries has not significantly decreased, based on a hypothesis of independence (p=0.116). A Chi^2 analysis of these data suggests that the number of injuries significantly decreased (Chi^2=3.83, p=0.05). In this low-frequency case, however, the Chi^2 is unreliable.

Note that a change from 6 injuries to 1 may be significant for other, non-statistical reasons. This is merely a reporting of statistical issues.

	No injuries	Injuries
2000-2003	207	6
2004-2007	210	1

Figure 11. The cumulative number of training events at an installation that were injury free versus training events that resulted in injury. Note that these data are invented for the purposes of this paper.

McNemar Test for Permanent Plots

As Chi^2 is analogous to the t-test for independent samples, the McNemar Test is analogous to a t-test on permanent (i.e., paired) samples. Do not use a Chi^2 with permanent plots, as the test's assumption of independence of observations is violated because of the spatial relationship of observations among years.

Imagine data on the presence of Species X with repeated measures in permanent plots (**Figure 12)**. Now, the data are more specific, since it is known what happened at individual plots. This example differs from the above examples, as it tests for relationships among data collected multiple times in the same individual plots, rather than testing for relationships among data summarized into categories over many different plots.

Tip: This whole table can be input for analysis at
http://www.fon.hum.uva.nl/Service/Statistics/McNemars_test.html

	Year 2 with	Year 2 without
Yr 1 Plots with Sp. X	130[a]	2[b]
Yr 1 Plots w/out Sp. X	4[c]	42[d]

Figure 12. A contingency table for the calculation of McNemar Chi^2 test.

The cells labeled b and c are the plots that changed: four plots gained Species X and two plots lost Species X

Under a null hypothesis of no effect of year, one expects that as many plots would *gain* species X as *lose* it. So, the expected frequencies of these cells is:

$$\frac{(b+c)}{2} = \frac{6}{2} = 3$$

Therefore, McNemar Chi^2 =

$$\frac{(2-3)^2}{3} + \frac{(4-3)^2}{3} = 0.667$$

This McNemar (paired sample) test statistic is distributed as a Chi^2, and can be looked up as a regular Chi^2 table (see Appendix 1).

The critical value for Chi^2 with p=0.05 and df=1 (**Appendix 1**) is 3.841. The observed Chi^2 = 0.667 is lower than this critical value and suggests that Species X did not change frequency in these plots. Note that the exact probability, calculated using a statistics package, of obtaining this result if the null hypothesis is true is $p = 0.688$.

Summary of Issues in the Analysis of Categorical Data

- Frequency and contingency table analysis is (or should be) very common in ecological and RTLA monitoring.
- There are several tests of association and independence of frequencies. Chi^2 is the most common one.
- Use the Chi^2 or G test for temporary plots.
- Beware of tables that contain small expected frequencies (< 5) in more than 20% of the cells; the test is erratic in such cases.
- A solution for analyzing data with low cell frequencies is Fisher's Exact Test.

- Another solution is to change the data collection (especially plot size) to change the expected frequencies. Obviously this solution is only possible before data collection begins.

- A post-sample solution to this problem can be to rescale the data (i.e., lump categories).

- An assumption of the Chi^2 test is *independence of observations.*

- The McNemar Test is a Chi^2 test for permanent plots when the assumption of independence of observations is violated.

Web Resources

http://udel.edu/~mcdonald/statgtestind.html
Provides Excel spreadsheets that perform a variety of tests, including Chi^2 and G tests for up to 10 columns and 50 rows.

http://www.fon.hum.uva.nl/Service/Statistics.html
Calculates a test of independence for frequency data, Chi^2 calculations and probability lookups.

http://www.dssresearch.com/toolkit/sscalc/size_p2.asp
Power analysis for a 2x2 G test and Chi^2.

http://department.obg.cuhk.edu.hk/researchsupport/statstesthome.asp
Contingency table analysis, such as Chi^2. Follow the links "Statistic Toolbox" and then "Statistical Tests".

http://www.dssresearch.com/toolkit/default.asp
Website that performs sample size calculations and power analysis for continuous and frequency data.

http://udel.edu/~mcdonald/statfishers.html
Provides an Excel-based calculator for Fisher's Exact Test in 2x2 tables.

http://www.fon.hum.uva.nl/Service/Statistics/McNemars_test.html
Website that performs McNemar's test in 2x2 tables, used for frequency data in permanent plots.

http://www.fon.hum.uva.nl/Service/Statistics.html
Chi^2 lookup table can also be used at the following site, just by inputting the numbers.

http://faculty.vassar.edu/lowry/abc.html
Analysis of three-way contingency tables.

References

Fisher, R.A. 1922. On the interpretation of chi^2 from contingency tables, and the calculation of P. Journal of the Royal Statistical Society **85**(1): 87-94.

Sokal, R. R. and F. J. Rohlf. 1981. Biometry, W. H. Freeman and Co., New York, N.Y.

Zar, J. H. 1996. Biostatistical Analysis. Third Edition. Prentice-Hall, Upper Saddle River, N.J.

Appendix 1: Critical values of chi^2 distribution with DF degrees of freedom
Look up the critical value for the appropriate df and the desired significance level. If the calculated Chi2 is greater than critical value, then conclude that there are non-random (i.e., significant) associations with the table.

	Probability of exceeding the critical value				
DF	**0.1**	**0.05**	**0.025**	**0.1**	**0.001**
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
6	10.645	12.592	14.449	16.812	22.458
7	12.017	14.067	16.013	18.475	24.322
8	13.362	15.507	17.535	20.090	26.125
9	14.684	16.919	19.023	21.666	27.877
10	15.987	18.307	20.483	23.209	29.588
11	17.275	19.675	21.920	24.725	31.264
12	18.549	21.026	23.337	26.217	32.910
13	19.812	22.362	24.736	27.688	34.528
14	21.064	23.685	26.119	29.141	36.123
15	22.307	24.996	27.488	30.578	37.697
16	23.542	26.296	28.845	32.000	39.252
17	24.769	27.587	30.191	33.409	40.790
18	25.989	28.869	31.526	34.805	42.312
19	27.204	30.144	32.852	36.191	43.820
20	28.412	31.410	34.170	37.566	45.315
21	29.615	32.671	35.479	38.932	46.797
22	30.813	33.924	36.781	40.289	48.268
23	32.007	35.172	38.076	41.638	49.728
24	33.196	36.415	39.364	42.980	51.179
25	34.382	37.652	40.646	44.314	52.620
26	35.563	38.885	41.923	45.642	54.052
27	36.741	40.113	43.195	46.963	55.476
28	37.916	41.337	44.461	48.278	56.892
29	39.087	42.557	45.722	49.588	58.301
30	40.256	43.773	46.979	50.892	59.703
31	41.422	44.985	48.232	52.191	61.098
32	42.585	46.194	49.480	53.486	62.487
33	43.745	47.400	50.725	54.776	63.870
34	44.903	48.602	51.966	56.061	65.247
35	46.059	49.802	53.203	57.342	66.619
36	47.212	50.998	54.437	58.619	67.985

37	48.363	52.192	55.668	59.893	69.347
38	49.513	53.384	56.896	61.162	70.703
39	50.660	54.572	58.120	62.428	72.055
40	51.805	55.758	59.342	63.691	73.402
41	52.949	56.942	60.561	64.950	74.745
42	54.090	58.124	61.777	66.206	76.084
43	55.230	59.304	62.990	67.459	77.419
44	56.369	60.481	64.201	68.710	78.750
45	57.505	61.656	65.410	69.957	80.077
46	58.641	62.830	66.617	71.201	81.400
47	59.774	64.001	67.821	72.443	82.720
48	60.907	65.171	69.023	73.683	84.037
49	62.038	66.339	70.222	74.919	85.351
50	63.167	67.505	71.420	76.154	86.661
51	64.295	68.669	72.616	77.386	87.968
52	65.422	69.832	73.810	78.616	89.272
53	66.548	70.993	75.002	79.843	90.573
54	67.673	72.153	76.192	81.069	91.872
55	68.796	73.311	77.380	82.292	93.168
56	69.919	74.468	78.567	83.513	94.461
57	71.040	75.624	79.752	84.733	95.751
58	72.160	76.778	80.936	85.950	97.039
59	73.279	77.931	82.117	87.166	98.324
60	74.397	79.082	83.298	88.379	99.607
61	75.514	80.232	84.476	89.591	100.888
62	76.630	81.381	85.654	90.802	102.166
63	77.745	82.529	86.830	92.010	103.442
64	78.860	83.675	88.004	93.217	104.716
65	79.973	84.821	89.177	94.422	105.988
66	81.085	85.965	90.349	95.626	107.258
67	82.197	87.108	91.519	96.828	108.526
68	83.308	88.250	92.689	98.028	109.791
69	84.418	89.391	93.856	99.228	111.055
70	85.527	90.531	95.023	100.425	112.317
71	86.635	91.670	96.189	101.621	113.577
72	87.743	92.808	97.353	102.816	114.835
73	88.850	93.945	98.516	104.010	116.092
74	89.956	95.081	99.678	105.202	117.346
75	91.061	96.217	100.839	106.393	118.599
76	92.166	97.351	101.999	107.583	119.850
77	93.270	98.484	103.158	108.771	121.100
78	94.374	99.617	104.316	109.958	122.348
79	95.476	100.749	105.473	111.144	123.594

80	96.578	101.879	106.629	112.329	124.839
81	97.680	103.010	107.783	113.512	126.083
82	98.780	104.139	108.937	114.695	127.324
83	99.880	105.267	110.090	115.876	128.565
84	100.980	106.395	111.242	117.057	129.804
85	102.079	107.522	112.393	118.236	131.041
86	103.177	108.648	113.544	119.414	132.277
87	104.275	109.773	114.693	120.591	133.512
88	105.372	110.898	115.841	121.767	134.746
89	106.469	112.022	116.989	122.942	135.978
90	107.565	113.145	118.136	124.116	137.208
91	108.661	114.268	119.282	125.289	138.438
92	109.756	115.390	120.427	126.462	139.666
93	110.850	116.511	121.571	127.633	140.893
94	111.944	117.632	122.715	128.803	142.119
95	113.038	118.752	123.858	129.973	143.344
96	114.131	119.871	125.000	131.141	144.567
97	115.223	120.990	126.141	132.309	145.789
98	116.315	122.108	127.282	133.476	147.010
99	117.407	123.225	128.422	134.642	148.230
100	118.498	124.342	129.561	135.807	149.449